ACHIEVING A MAXIMUM PROFIT UNDER AN OPTIMAL USE OF MACHINES CAPACITY

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Abstract: *The purpose of the paper is to present how to use sensitivity analysis tools of the optimal solution to the variation coefficients of the linear programming model. Using the Linear and Integer Programming module of the WinQSB software, several tests have been conducted such as: sensitivity analysis of the current optimal solution to the variation of a single coefficient from the objective function, sensitivity analysis of the current shadow prices to the variation of a single free term and parameterization of a single coefficient of the objective function, as is of a single free term.*

Key words: *modeling, linear programming, objective function, profit, sensitivity analysis.*

INTRODUCTION

The scientific study of a system or phenomenon can be done by real or artificial experimentation. In the economic domain, real experimentation is rare because it involves high costs and risks, while artificial experimentation, and allows avoiding real situations with sometimes catastrophic implications.

Analysis of complex economic systems can be made with the analytical methods and techniques to solve real economic models.

Among the models used in the production problems include the models for economic problems that lead to linear programming problems such as:

- Models for problems using limited resources efficiently;
- Models for the problem of optimal use of machines capacity;
- Models for mixture problems;
- Models of investments;
- Models of cutting.

This paper presents only one type of economic model namely, the model of the optimal use of production capacity problem which refers to the operative programming of production, in the conditions of limited production capacity with the aim to maximize profits. Restrictions refer to a series of machines that execute the desired products, b_i being the available time of the machine i on the analyzed period and a_{ij} the required processing time of a type j product on the machine i. The aim is to determine the quantities that need to be achieved x_j from the type of products P_j , so that the total profit is maximum.

The general mathematical model of such a problem will be:

$$[\max]f = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$$

$$\sum_{j=1}^{n} a_{ij}x_j \le b_i, \qquad i = \overline{1,m}$$

$$x_j \ge 0; \quad j = \overline{1,n}$$

If instead of making profit an enterprise would consider minimizing expenses or the losses of the enterprise concerning the making of those products, modelling at the level of restrictions would be identical but the objective function will be of minimal type.

The models for determining the production capacities are of a big variety, therefore they can be very complex. The production capacity is determined on the fund base of the available time of the machines and can be optimized from many points of view, namely:

reducing consumption of material and human resources and their optimal exploitation, depending on the conditions of realizing the contracted sortimental program and of some minimal costs.

PROBLEM DATA

All the models that are part of economic problems category which lead to linear programming problems can be solved using the Linear and Integer Programming module. In this paper, we will approach only a model for the problem of optimal use of machine capacity.

The terms of the application:

It is assumed that in a section of an enterprise three products are manufactured P_1, P_2, P_3 for which are needed four operations to execute done by four machines M_1, M_2, M_3, M_4. Time required for the four operations corresponding to each product are given in the following table.

Table 1

Products \ Machines	M_1	M_2	M_3	M_4	Observation
P_1	10	7	11	12	Time is measured in minutes
P_2	5	11	14	13	
P_3	12	8	15	10	

In the production process must be taken into account by:

1) Machine M_1 can function monthly at most 180 hours;
2) Machine M_2 must be used at maximum, 208 hours;
3) Machine M_3 must be used at maximum, 160 hours;
4) Machine M_4 must function at least 120 and at most 175 hours;
5) From certain economic reasons product P_2 must take a share of at least 30% and at most 60% of the total monthly production.

Achieving the established products, the section can have for P_1 a unitary profit of 115 m.u, for P_2 of 142 m.u., for P_3 of 97 m.u. It is required that by respecting the technical standards or imposed plan to achieve a maximum profit.

PROBLEM SOLVING

After making the necessary notations namely: x_1, x_2, x_3 the number of units from the products P_1, P_2, P_3 that need to be attained, we obtain the following restrictions of the mathematical model:

$$\begin{cases} 10x_1 + 5x_2 + 12x_3 \le 10.800 \\ 7x_1 + 11x_2 + 8x_3 \le 12.480 \\ 11x_1 + 14x_2 + 15x_3 \le 9.600 \\ 12x_1 + 13x_2 + 10x_3 \le 10.500 \\ 12x_1 + 13x_2 + 10x_3 \ge 7.200 \\ \frac{30}{100}(x_1 + x_2 + x_3) \le x_2 \le \frac{60}{100}(x_1 + x_2 + x_3) \end{cases}$$

Slightly adjusted, the above relation becomes the following relation:

$$\begin{cases} 10x_1 + 5x_2 + 12x_3 \le 10.800 \\ 7x_1 + 11x_2 + 8x_3 \le 12.480 \\ 11x_1 + 14x_2 + 15x_3 \le 9.600 \\ 12x_1 + 13x_2 + 10x_3 \le 10.500 \\ 12x_1 + 13x_2 + 10x_3 \ge 7.200 \\ 3x_1 - 7x_2 + 3x_3 \le 0 \\ 6x_1 - 4x_2 + 6x_3 \ge 0 \\ x_j \ge 0, \quad j = 1,2,3 \end{cases}$$

The total profit of the section will be given by:

FOB: $\max[Z(x)] = 115\,x_1 + 142\,x_2 + 97\,x_3$

Following the entry of input data, we obtain the following table, where the optimal solution of the problem is shown (Fig.1), or may opt for a brief solution (Fig. 2).

The solution provided, specifies that to get maximum profit, 300 P_1 parts must be manufactured and 450 P_2. Therefore the profit that would be obtained would be of 98.400 m.u. Manufacturing of a P_3 units, would involve a decrease of total profit by 59 m.u.

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
X1	300,0000	115,0000	34.500,0000	0	basic	111,5714	M
X2	450,0000	142,0000	63.900,0000	0	basic	-76,6667	146,3636
X3	0	97,0000	0	-59,0000	at bound	-M	156,0000
Objective	Function	(Max.) =	98.400,0000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
C1	5.250,0000	<=	10.800,0000	5.550,0000	0	5.250,0000	M
C2	7.050,0000	<=	12.480,0000	5.430,0000	0	7.050,0000	M
C3	9.600,0000	<=	9.600,0000	0	10,2500	7.314,2860	10.666,6700
C4	9.450,0000	<=	10.500,0000	1.050,0000	0	9.450,0000	M
C5	9.450,0000	>=	7.200,0000	2.250,0000	0	-M	9.450,0000
C6	-2.250,0000	<=	0	2.250,0000	0	-2.250,0000	M
C7	0	<=	0	0	0,3750	-2.742,8570	2.420,1680

Fig.1. Combined ratio – optimal solution of the problem

Decision Variable	Solution Value	Unit Cost or Profit C(j)	Total Contribution	Reduced Cost	Basis Status
X1	300,0000	115,0000	34.500,0000	0	basic
X2	450,0000	142,0000	63.900,0000	0	basic
X3	0	97,0000	0	-59,0000	at bound
Objective	Function	(Max.) =	98.400,0000		

Fig. 2. Brief solution

SENSITIVITY ANALYSIS OF THE CURRENT OPTIMAL SOLUTION

If we make the sensitivity analysis of the current optimal solution to the variation of a single coefficient of the objective function, from the table in figure 1 the result is that if the selling price of the P_3 product, is lower than 156 m.u./unit, then x_3 (product quantity P_3) will remain zero. For example, the increase from 97 m.u./unit to 150 m.u./unit of the selling price of the product P_3 will not generate any change in regards to the optimal profit of the enterprise. But for the entry to be efficient in the production program of the P_3

product, it is necessary that its selling price to be 156 m.u., that is (97+59) m.u. Then we have the situation presented in figure 3.

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X1	12,0000	115,0000	1.380,0000	0	basic
2	X2	402,0000	142,0000	57.084,0000	0	at bound
3	X3	256,0000	156,0000	39.936,0000	0	basic
	Objective	Function	(Max.) =	98.400,0000		
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	C1	5.202,0000	<=	10.800,0000	5.598,0000	0
2	C2	6.554,0000	<=	12.480,0000	5.926,0000	0
3	C3	9.600,0000	<=	9.600,0000	0	10,2500
4	C4	7.930,0000	<=	10.500,0000	2.570,0000	0
5	C5	7.930,0000	>=	7.200,0000	730,0000	0
6	C6	-2.010,0000	<=	0	2.010,0000	0
7	C7	0	<=	0	0	0,3750

Fig.3. Optimal solution offered in the case of modifying the price of the product P3

Therefore, this modified problem admits a unique finite optimal solution. The maximum profit that the enterprise can obtain is also by 98.400 u.m. In this case, the optimal combination of products consists of 12 units of the product P_1, 402 units of P_2 and 256 P_3 units.

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	300,0000	120,0000	36.000,0000	0	basic	111,5714	M
2	X2	450,0000	142,0000	63.900,0000	0	basic	-80,0000	152,7273
3	X3	0	97,0000	0	-64,6250	at bound	-M	161,6250
	Objective	Function	(Max.) =	99.900,0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	5.250,0000	<=	10.800,0000	5.550,0000	0	5.250,0000	M
2	C2	7.050,0000	<=	12.480,0000	5.430,0000	0	7.050,0000	M
3	C3	9.600,0000	<=	9.600,0000	0	10,4063	7.314,2860	10.666,6700
4	C4	9.450,0000	<=	10.500,0000	1.050,0000	0	9.450,0000	M
5	C5	9.450,0000	>=	7.200,0000	2.250,0000	0	-M	9.450,0000
6	C6	-2.250,0000	<=	0	2.250,0000	0	-2.250,0000	M
7	C7	0	<=	0	0	0,9219	-2.742,8570	2.420,1680

Fig.4.

Continuing with the same sensitivity analysis from above, and modifying the cost to one of the products that are part of the optimal combination presented in figure 1, meaning P_1 or P_2, the value of the objective function will be modified. If the cost of the product P_1 will increase to 120 m.u./unit, the objective function will take the value of 99.900 m.u. (fig.4), and if the cost of the product will decrease to 112 m.u./unit (111,5714 being the minimum possible value), the objective function will reach to 97.500 m.u. (fig.5).

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	300,0000	112,0000	33.600,0000	0	basic	111,5714	M
2	X2	450,0000	142,0000	63.900,0000	0	basic	-74,6667	142,5455
3	X3	0	97,0000	0	-55,6250	at bound	-M	152,6250
	Objective	Function	(Max.) =	97.500,0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	5.250,0000	<=	10.800,0000	5.550,0000	0	5.250,0000	M
2	C2	7.050,0000	<=	12.480,0000	5.430,0000	0	7.050,0000	M
3	C3	9.600,0000	<=	9.600,0000	0	10,1563	7.314,2860	10.666,6700
4	C4	9.450,0000	<=	10.500,0000	1.050,0000	0	9.450,0000	M
5	C5	9.450,0000	>=	7.200,0000	2.250,0000	0	-M	9.450,0000
6	C6	-2.250,0000	<=	0	2.250,0000	0	-2.250,0000	M
7	C7	0	<=	0	0	0,0469	-2.742,8570	2.420,1680

Fig.5.

PARAMETRIC ANALYSIS OF A SINGLE COEFICIENT OF THE OBJECTIVE FUNCTION

After the parametric analysis of the coefficient c_3, associated to the variable x_3, the following results are obtained, presented in figure 6.

Range	From Coeff. of X3	To Coeff. of X3	From OBJ Value	To OBJ Value	Slope	Leaving Variable	Entering Variable
1	97,0000	156,0000	98.400,0000	98.400,0000	0	X1	X3
2	156,0000	M	98.400,0000	M	266,6667		
3	97,0000	-M	98.400,0000	98.400,0000	0		

Fig.6.

Two intervals were obtained at the right of the initial price of 97 m.u./unit and a single interval at its left. In figure 7 the variation of the objective function is shown, namely the maximum profit that can be achieved if the prices of the P_3 are modified, and the others remain unchanged.

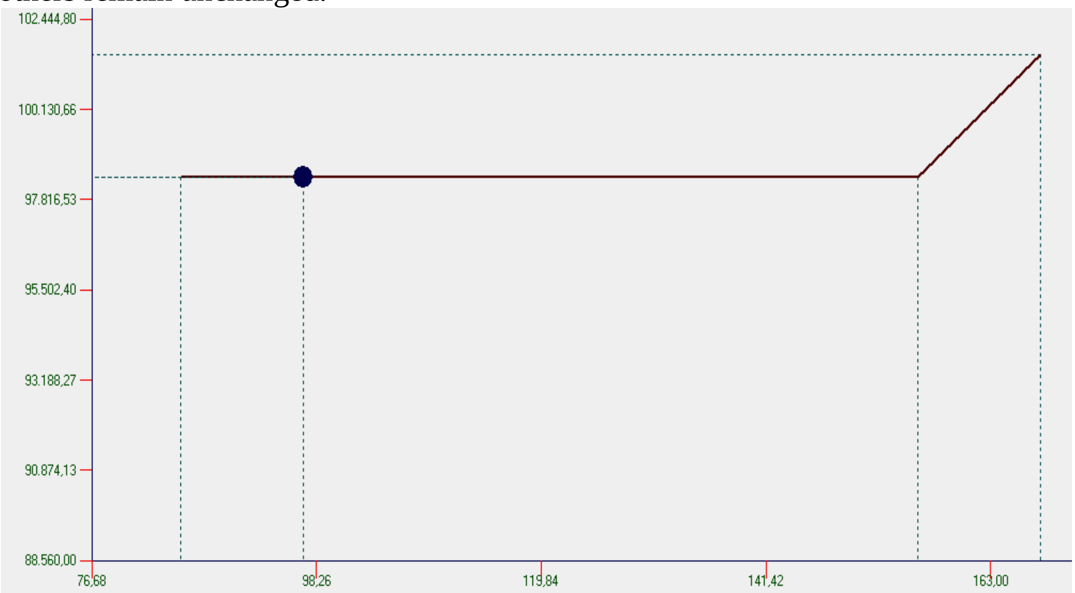

Fig.7. Variation of the objective function

SENSITIVITY ANALYSIS OF THE CURRENT SHADOW PRICES

To achieve the sensitivity analysis of the current shadow prices to the variation of a single term in the right side of restrictions, it also starts from the initial optimal solution, still the products P_1 and P_2 will be made, but in other quantities.

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X1	312,0000	115,0000	35.880,0000	0	basic
2	X2	469,0000	142,0000	66.598,0000	0	basic
3	X3	0	97,0000	0	97,0000	at bound
	Objective	Function	(Max.) =	102.478,0000		
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	C1	5.465,0000	<=	10.800,0000	5.335,0000	0
2	C2	7.343,0000	<=	12.480,0000	5.137,0000	0
3	C3	9.998,0000	<=	10.000,0000	2,0000	0
4	C4	9.841,0000	<=	10.500,0000	659,0000	0
5	C5	9.841,0000	>=	7.200,0000	2.641,0000	0
6	C6	-2.347,0000	<=	0	2.347,0000	0
7	C7	-4,0000	<=	0	4,0000	0

Fig.8. Results of the sensitivity analysis

The shadow price associated to the C_3 restriction, regarding the functioning minutes of the M_3 machine is 10,25. This price varies between 7.314,2860 and 10.666,67. If the availability of the resource increases the current quantity from 9.600 to 10.000, then we obtain the results from figure 8.

PARAMETRIC ANALYSIS OF A FREE TERM

The results of the sensitivity analysis for the variation of the free term b_3, related to the restriction C_3, are presented in figure 9.

Range	From RHS of C3	To RHS of C3	From OBJ Value	To OBJ Value	Slope	Leaving Variable	Entering Variable
1	9.600,0000	10.666,6700	98.400,0000	109.333,3000	10,2500	Slack_C4	Slack_C7
2	10.666,6700	11.307,6900	109.333,3000	114.692,3000	8,3600	X1	Slack_C3
3	11.307,6900	M	114.692,3000	114.692,3000	0		
4	9.600,0000	7.314,2860	98.400,0000	74.971,4300	10,2500	Surplus_C5	
5	7.314,2860	-Infinity	Infeasible				

Fig.9 Results of the parametric analysis

Five intervals of variation were obtained for the functioning time of the machine M_3: four to the right and an interval in its left.

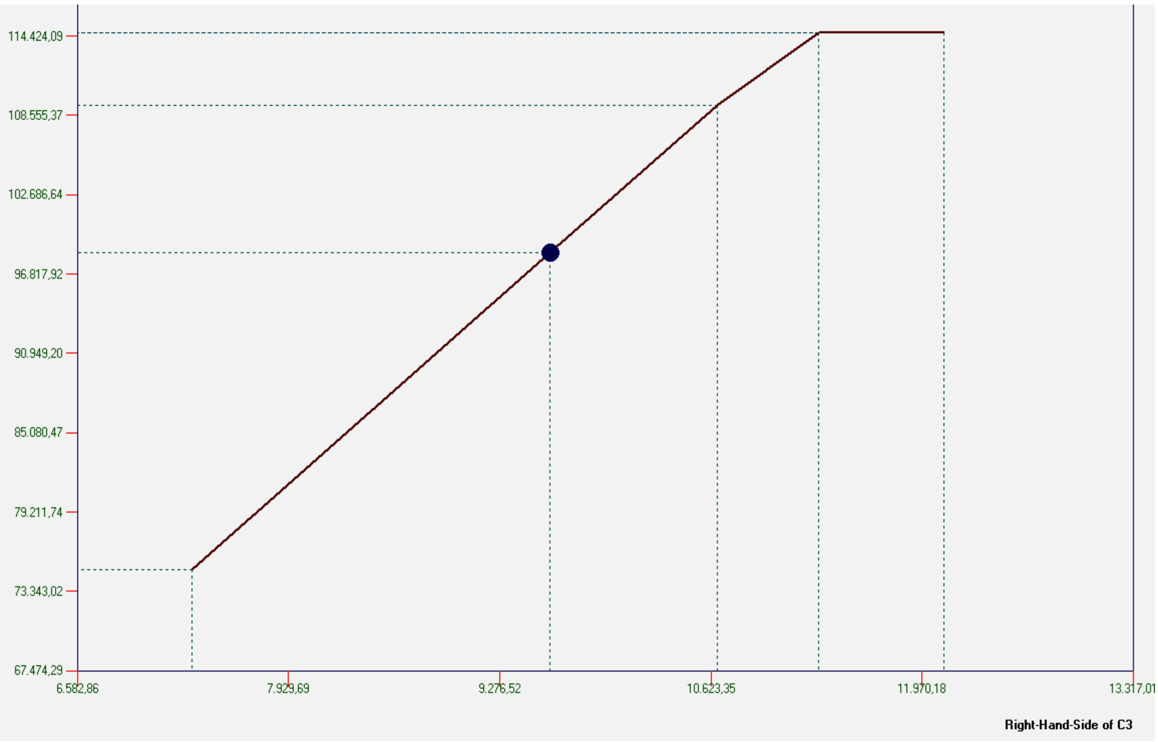

Fig.10 Variation of the maximum profit

In the Slope column of the table from figure 8, for each variation interval of the free term b_3, is shown the ratio between the variation of the objective function and the variation of the free term.

In figure 10 the variation of the maximum profit is shown that can be achieved if the functioning time of the machine M_3 is modified, and the other free terms remain unchanged.

CONCLUSIONS

The theory of linear programming offers a variety of instruments of sensitivity analysis of optimal solution to the variation of the coefficients of the linear programming model. After obtaining the optimal solution, before implementing it in practice, it is advised to perform sensitivity analysis in the conversational system.

Of these, in this paper were conducted:

- sensitivity analysis of current optimal solution to the variation of a single coefficient from the objective function;
- parameterization of a single coefficient from the objective function;
- sensitivity analysis of the current shadow prices to the variation of a single free term;
- parameterization of a single free term.

REFERENCES

1. AMARIEI, OLGA IOANA, 2009, Aplicații ale programului WinQSB în simularea sistemelor de producție, Eftimie Murgu Press, Reşița, ISBN 978-973-1906-16-4.

2. GILLICH NICOLETA, ANGHEL CORNELIA, AMARIEI, OLGA IOANA, 2009, Cercetări operaționale. Teorie şi aplicații, Eftimie Murgu Press, Reşița, ISBN 978-973-1906-43-0.

3. BLĂJINĂ, OVIDIU AURELIAN, 2006, Produse software aplicate în programarea matematică şi teoria jocurilor. Albastră Press, Cluj-Napoca.
4. BLĂJINĂ, OVIDIU AURELIAN, 2011, „Decizii optime în management cu WinQSB 2.0" Albastră Press, Cluj-Napoca.
5. RAȚIU–SUCIU CAMELIA, LUBAN FLORICA ş.a., 2009, Modelare economică. Ediția a doua, ASE Press Bucureşti.
6. RAȚIU–SUCIU CAMELIA, LUBAN FLORICA ş.a., 2002, Modelare economică aplicată. 50 Studii de caz. 525 Teste, Economică Press, Bucureşti.